SUBJECT CODE	SUBJECT	PAPER
A-15-03	MATHEMATICAL SCIENCES	III

HALL TICKET NUMBER									QUESTION BOOKLET NUMBER
									601622

OMR SHEET NUMBER					

DURATION	MAXIMUM MARKS	NUMBER OF PAGES	NUMBER OF QUESTIONS
2 HOUR 30 MINUTES	150	16	75

This is to certify that, the entries made in the above portion are correctly written and verified.

Candidate's Signature | **Name and Signature of Invigilator**

Instructions for the Candidates

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested **to open the booklet and compulsorily examine it as below**:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example: (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Answer Sheet given to you.** If you mark at any place other than in the circle in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. **The candidate must handover the OMR Answer Sheet to the invigilators at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. The candidate is allowed to take away the carbon copy of OMR Sheet and used Question paper booklet at the end of the examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము డెబ్బైఐదు బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో <u>ఈ ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా పరిచూసుకోండి.</u>
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్‌పేజి అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నాపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నాపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నాపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్దిష్టస్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతిప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి.
   ఉదాహరణ : (A) (B) ● (D)
   (C) సరైన ప్రతిస్పందన అయితే
5. ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
8. OMR పత్రము పై నిర్ణీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. పరీక్ష పూర్తయిన తర్వాత మీ **OMR** పత్రాన్ని తప్పనిసరిగా పరీక్ష పర్యవేక్షకుడికి ఇవ్వాలి. వాటిని పరీక్ష గది బయటకు తీసుకువెళ్ళకూడదు. పరీక్ష పూర్తయిన తరువాత అభ్యర్థులు ప్రశ్న పత్రాన్ని, OMR పత్రం యొక్క కార్బన్ కాపీని తీసుకువెళ్ళవచ్చు.
10. నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.
11. లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్షాగదిలో ఉపయోగించడం నిషేధం.
12. తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.

MATHEMATICAL SCIENCES

Paper – III

1. If $f:[0, 1] \to \mathbb{R}$ is defined by

$$f(x)=\begin{cases} 0 & \text{if } x=0 \\ \frac{1}{2^{r-1}} & \text{if } \frac{1}{2^r}<x\leq\frac{1}{2^{r-1}} \text{ for } r=1, 2, 3, \dots \end{cases}$$

then $\int_0^1 f(x)\,dx =$

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$

(C) $\frac{4}{5}$ (D) $\frac{5}{6}$

2. The correct statement among the following is

(I) $f_n(x) = x^n$ converges uniformly to 0 on $[0, a)$ for any a with $0 < a < 1$

(II) $f_n(x)=\frac{nx}{1+n^2x^2}$ converges uniformly to 0 on $\mathbb{R}$

(III) $f_n(x)=x^{n-1}(1-x)$ converges only pointwise on (0, 1)

(IV) $f_n(x) = x^n$ converges uniformly on [0, 1)

(A) I (B) II

(C) III (D) IV

3. The improper integral $\int_0^\infty \frac{x^m}{1+x^n}dx$ converges only if

(A) $n > m + 1$ (B) $n = m$

(C) $n \leq m+1$ (D) $n \neq m$

4. If $f:(a, b) \to \mathbb{R}$ is monotonic increasing function and $a < c < b$ then $f(c - 0)$, the left hand limit of f at c is equal to

(A) $\inf\{f(t) : c < t < b\}$

(B) $\inf\{f(t) : a < t < c\}$

(C) $\sup\{f(t) : a < t < c\}$

(D) $\sup\{f(t) : c < t < b\}$

5. If $\mathfrak{M}$ is the collection of all Lebesgue measurable sets in $\mathbb{R}$ then the incorrect statement among the following is :

(A) $\mathfrak{M}$ is σ - algebra of subsets of $\mathbb{R}$

(B) Every open set in $\mathbb{R}$ is a member of $\mathfrak{M}$

(C) Every closed set in $\mathbb{R}$ is a member of $\mathfrak{M}$

(D) Every member of $\mathfrak{M}$ is a Borel set

6. The function

$$f(x, y)=x^3+y^3-6(x^2+y^2)+12xy-75(x+y)$$

has maximum value at the point

(A) (5, 5) (B) (–5, –5)

(C) (1, 7) (D) (7, 1)

7. In the metric space $\mathbb{R}^2$, the Euclidean plane, consider the lists given below :

List - I	List - II
(a) A closed unbounded set	(1) $\{(\frac{1}{n}, 0) \in \mathbb{R}^2 : n = 1, 2, 3, ...\}$
(b) A bounded set which is not closed	(2) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 7\}$
(c) An unbounded set which is both open and closed	(3) ϕ
(d) A non-empty compact set	(4) $\{(0, n) \in \mathbb{R}^2 : n = 0, \pm 1, \pm 2, ...\}$
	(5) $\{(x, y) \in \mathbb{R}^2 : (x+1)^2 + y^2 > 4\}$
	(6) $\mathbb{R}^2$

The correct matching of List-I from List-II is

	(a)	(b)	(c)	(d)
(A)	3	2	6	1
(B)	4	3	5	2
(C)	4	1	6	2
(D)	3	2	4	1

8. **Assertion (A) :** In the normed linear space $\mathscr{C}$ [a, b] of continuous real-valued functions on [a, b] with the supremum norm, the set $\mathscr{P}$ of all polynomials is a dense subset

Reason (R): Weierstrauss approximation theorem holds

(A) (A) is true but (R) is false

(B) Both (A) and (R) are false

(C) Both (A) and (R) are true but (R) is not a correct explanation for (A)

(D) Both (A) and (R) are true; and (R) is the correct explanation for (A)

9. The function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ has minimum at the point

(A) (1, 2) (B) (–1, 0)

(C) (1, 0) (D) (–1, –2)

10. The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

(A) $\frac{\pi}{2}$ (B) 2π

(C) π (D) $\frac{3\pi}{2}$

11. The determinant of the matrix

$$A=\begin{pmatrix} 1 & 3 & -3 \\ -3 & -5 & 2 \\ -4 & 4 & -6 \end{pmatrix} \text{ is}$$

(A) 48 (B) 40

(C) 38 (D) 50

12. Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(a_1, a_2)=(a_1+3a_2,\ 0,\ 2a_1-4a_2)$$

Let β and γ be the standard ordered basis for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Then the matrix corresponding $[T]_\beta^\gamma$ is

(A) $\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -4 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ 0 & 1 \\ 2 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ -2 & -4 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 3 \\ 0 & 0 \\ -2 & -4 \end{bmatrix}$

13. Let the matrix $A=\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ and $I=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then $A^2-2A+5I$ is equal to

(A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

14. The solution of the system of equations

$x_1+2x_2-x_3=-1$

$2x_1+2x_2+x_3=1$

$3x_1+5x_2-2x_3=-1$ is

(A) $\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$

15. The rank of the matrix $A=\begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{pmatrix}$ is

(A) 1 (B) 3

(C) 0 (D) 2

16. Let f (z) be analytic in $|z|\le 1$ and $|f(z)|\le 1$ with $f(0)=\frac{1+i}{\sqrt{2}}$. Then f (i) – f (1) =

(A) 0 (B) $i-1$

(C) i (D) 1

17. $\int_0^\pi e^{\cos\theta}\cos(\sin\theta)\,d\theta =$

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

18. Suppose C is the circle $|z| = 2$ positively oriented. Then $\int_C \frac{1}{z^2+2iz-1} dz =$

(A) $2\pi i$ (B) -2π

(C) 2π (D) 0

19. Suppose f (z) is analytic on $|z| \leq 1$ such that $|f(z)-z| < |z|$ on $|z| = 1$. Then the number of zeros of f (z) in $|z| < 1$ is

(A) 1 (B) 2

(C) 3 (D) 5

20. $\int_0^\infty \frac{\sin 5x}{x} dx =$

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

21. Suppose C is the circle $|z| = 1$ positively oriented, and $f(z) = \frac{e^z}{z(z-2)(z-3)...(z-9)}$.

Then $\int_C f(z) dz =$

(A) $\frac{-\pi i}{9!}$ (B) $\frac{\pi i}{9!}$

(C) $\frac{-2\pi i}{9!}$ (D) $\frac{2\pi i}{9!}$

22. The number of groups G such that $\left|\frac{G}{Z(G)}\right| = 119$, where z (G) is the centre of G, is

(A) 1 (B) 0

(C) 2 (D) infinite

23. Let $\alpha = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ and $\beta = \sin\frac{2\pi}{5}$. Then the degree of the field $Q(\alpha)$ over $Q(\beta)$ is

(A) 1 (B) 2

(C) 4 (D) infinite

24. The order of Galois group of $x^4 + x^2 + 1$ over Q is

(A) 1 (B) 3

(C) 2 (D) 5

25. Let F, K be fields such that $K \subseteq F$ and $u \in F$. If $[K(u):K] = 5$ then $[K(u^2):K] =$

(A) 25 (B) 5

(C) 10 (D) 15

26. Consider $\mathbb{Z}[i]$ the ring of Gaussian integers and the maximal ideal $M = \{a+bi : 3|a,\ 3|b\}$ in $\mathbb{Z}[i]$. Then the order of the quotient ring $\frac{R}{M}$ is

(A) 3 (B) 5

(C) 7 (D) 9

27. A maximal ideal in $\mathbb{R}[x]$ among the following is

(A) $\langle x^4+4\rangle$ (B) $\langle x^3+1\rangle$

(C) $\langle x^5+1\rangle$ (D) $\langle x^2+2\rangle$

28. Let T be the Cantor's set in $\mathbb{R}$. Then which of the following is incorrect ?

(A) T is closed

(B) T is compact

(C) T is bounded

(D) T is connected

29. Consider the topology

$T=\{\phi, X, \{x\}, \{z, w\}, \{x, z, w\}, \{y, z, w, u\}\}$

on $X=\{x, y, z, w, u\}$. Then the number of components of X is

(A) 1 (B) 2

(C) 3 (D) 4

30. Suppose X is a compact metric space. Then which of the following statements is not true ?

(A) X is separable

(B) X is closed

(C) X is sequentially compact

(D) X is not separable

31. If the complementary function of the differential equation $y''-y'-6y=0$ is $y_c=Ae^{\alpha x}+Be^{\beta x}$, then $\alpha^2\beta^2=$

(A) 4 (B) 16

(C) 64 (D) 36

32. If the solution of the diferential equation $(D^3-D^2-4D+4)\,y=e^{3x}$ is $y=c_1e^x+c_2e^{2x}+c_3e^{-2x}+Ke^{3x}$, then K =

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) 5

(D) 10

33. If $y(0)=2$, $y'(0)=-1$ and $\frac{d^2y}{dx^2}+y=0$, then $y=$

(A) 2sinx + cosx (B) 2cosx + sinx

(C) 2cosx – sinx (D) 2sinx – cosx

34. The general solution of the differential equation

$\left(x^2z-y^3\right)dx+3xy^2dy+x^3dz=0$ is

(A) $x^2z^2+y^3=c$

(B) $x^2z+y^3=cx$

(C) $x^2z+y^3=c$

(D) $x^2z^2+y^3=cx$

35. If $x^2+y^2+lz^3+mz^2+2=0$ is the surface that intersects the system of surfaces $z(x+y)=C(3z+1)$ orthogonally and passes through the circle $x^2+y^2=1$, $z=1$ then $l+m=$

(A) 3 (B) –3

(C) 2 (D) –2

36. If $z=e^y$, $\frac{\partial z}{\partial x}=1$ when $x=0$, then the solution of $\frac{\partial^2 z}{\partial x^2}+z=0$ is $z=$

(A) $\sin x+e^y\cos x$

(B) $e^y\sin x+\cos x$

(C) $\sin x\cos x+e^y$

(D) $\sin x\cos x-e^y$

37. The partial differential equation obtained by eliminating a, b from the equation $z=xy+y\sqrt{x^2-a^2-b^2}$ is

(A) $px-qy=pq$ (B) $px+qy+pq=0$

(C) $px+qy=pq$ (D) $px-qy+pq=0$

38. If a cubic polynomial f (x) is such that f (0) = 1, f (1) = 0, f (2) = 1 and f (3) = 10, then f (4) =

(A) 43 (B) 33

(C) 23 (D) 13

39. With the standard notation $4\mu^2-\delta^2=$

(A) 4 (B) 3

(C) 2 (D) 1

40. By dividing [0, 1] into 4 equal sub intervals, the value of $\int_0^1 \frac{dx}{1+x}$ (using trapezoidal rule) correct to 3 decimals is

(A) 0.693 (B) 0.694

(C) 0.697 (D) 0.699

41. $\int_0^1 \left(y^2+\left(\frac{dy}{dx}\right)^2\right)dx$ along the path $y=x^2$ is

(A) $\frac{23}{15}$ (B) $\frac{18}{15}$

(C) $\frac{28}{15}$ (D) $\frac{33}{15}$

42. If the solution of

$$u'(x)+\int_0^1 \exp(x-y)u(y)dy=f(x),\ u(0)=0$$

is $u(x)=g(x)+\lambda\left(e^x-1\right)\int_0^1 e^{-y}g(y)dy$

where $g(x)=\int_0^x f(t)dt$, then $\lambda=$

(A) $\frac{e}{1+e}$

(B) $1-e$

(C) $\frac{-e}{1+e}$

(D) $\frac{1}{1-e}$

43. The resolvent kernel of

$$\varphi(x)=f(x)+\int_0^x(x-t)\varphi(t)dt \quad (t<x) \text{ is}$$

(A) sinh (x – y)

(B) sin (x – y)

(C) cos (x – y)

(D) cosh (x – y)

44. Assume that a piston executes a simple harmonic motion with an amplitude 0.15 m. If it passes through the centre of its motion with a speed of 0.3 m/s then the period of oscillation (in per seconds) is

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

45. If $\vec{F}=\left(ax+by^2\right)\vec{i}+c\,xy\,\vec{i}$ is conservative, then

(A) $a+b+c=0$

(B) $a^2=bc$

(C) $c=-2b$

(D) $b^2+c^2=2ab$

46. An event A is independent of itself if and only if

(A) P (A) = 0.5

(B) $P(A)+P(\overline{A})=1$

(C) P (A) = 0 or P (A) = 1

(D) P (A) = 0.25

47. Which of the following is not a property of a distribution function F (x) of a random variable X ?

(A) $P(a<X\le b)=F(b)-F(a)$

(B) $F(x)\le F(y)$ if $x<y$

(C) $F(-\infty)=0$ and $F(\infty)=1$

(D) $F(0)=\frac{1}{2}$

48. Let X be a random variable with mean μ and variance $\sigma^2>0$. Let $k>0$ be fixed. Then which of the following is not correct ?

(A) $P(|X-\mu|\ge k\sigma)\le\frac{1}{k^2}$

(B) $P(|X-\mu|\le k\sigma)\ge 1-\frac{1}{k^2}$

(C) $P(|X-\mu|\ge k)\le\frac{\sigma^2}{k^2}$

(D) $P(|X-\mu|\ge k)\le\frac{1}{k^2}$

49. Let $\{x_n\}$ be a sequence of i.i.d. random variables with finite mean μ and finite variance. Consider the following two statements :

$$P:\frac{2}{n(n+1)}\sum_{i=1}^{n} i\cdot x_i \xrightarrow{p} \mu;$$

$$Q:\frac{6}{n(n+1)(2n+1)}\sum_{i=1}^{n} i^2 x_i \xrightarrow{p} \mu$$

then which of the following is true ?

(A) Both P and Q are false

(B) Both P and Q are true

(C) P is true but Q is false

(D) P is false but Q is true

50. Let $\{x_n\}$ be a sequence of i.i.d. random variables with mean μ and finite variance. Let $\overline{X}_n$ be the mean of the first n random variables. Now, consider the statement

$$P: \frac{\sqrt{n}\left(\overline{X}_n - \mu\right)}{\sqrt{\overline{X}_n}} \xrightarrow{d} N(0, 1)$$

Then which of the following condition is necessary for P to be true ?

(A) X_1 follows Poisson P (μ)

(B) X_1 follows Binomial B (k, μ)

(C) X_1 follows Normal $N\left(\mu, \sigma^2\right)$

(D) X_1 follows Normal $N(\mu, 1)$

51. If Transition Probability Matrix (TPM) of a Markov chain (MC) is $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$, then its stationary distribution is given by

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(B) $\left(\frac{3}{4}, \frac{1}{4}\right)$

(C) $\left(\frac{3}{5}, \frac{2}{5}\right)$

(D) $\left(\frac{4}{5}, \frac{1}{5}\right)$

52. If TPM of a MC is $P = \begin{bmatrix} 0 \cdot 1 & 0 \cdot 2 & 0 \cdot 7 \\ 0 \cdot 2 & 0 \cdot 2 & 0 \cdot 6 \\ 0 \cdot 6 & 0 \cdot 1 & 0 \cdot 3 \end{bmatrix}$,

then $P\left(X_3 = 1 \middle| X_1 = 0\right)$ is

(A) 0.26

(B) 0.17

(C) 0.14

(D) 0.13

53. Let X be normal $N\left(\mu, \sigma^2\right)$. Let I be the set of all integers. Then $P(X \in I)$ is equal to

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $\frac{1}{\sqrt{2\pi}}$

54. Let $\overline{X}$ be the mean of n i.i.d. standard Cauchy random variables. Then $\overline{X}$ is distributed as

(A) Standard normal

(B) Laplace

(C) Standard Cauchy

(D) Cauchy with scale parameter $\frac{1}{\sqrt{n}}$

55. The procedure of improving the efficiency of an unbiased estimator with the use of a sufficient statistic was invented by

(A) Lehmann

(B) Scheffe

(C) Cramer

(D) C.R. Rao

56. Let $X_1, X_2, \cdots, X_n$ be a random sample of size n drawn from uniform $U(0, \theta)$ distribution. If $X_{(n)}$ is the largest observation in the sample, then which of the following is an unbiased estimator of θ ?

(A) Sample mean

(B) Sample median

(C) $X_{(n)}$

(D) $\frac{n+1}{n} X_{(n)}$

57. The efficiency of sample mean as compared to sample median as estimator of the mean of a normal population in percentage is

(A) 64 (B) 157

(C) 317 (D) 31.5

58. A random sample of size 1 is taken from a p.d.f.

$f(x, \theta) = \frac{2(\theta - x)}{\theta^2}, 0 < x < \theta\ ;\ f(0, \theta) = 0,$

elsewhere. The most powerful test of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1, \theta_1 < \theta_0$ at level α is given by

(A) $\phi(x) = 1$ if $x > \theta_0 \left[1 - \sqrt{1-\alpha}\right]$

(B) $\phi(x) = 1$ if $x < \theta_0 \left[1 - \sqrt{1-\alpha}\right]$

(C) $\phi(x) = 1$ if $x > \theta_0^\alpha$

(D) $\phi(x) = 1$ if $x < \theta_0^\alpha$

59. Let $X \sim N(\theta, \sigma^2)$. Which of the following is a simple hypothesis ?

(A) $H : \theta = \theta_0$ (B) $H : \sigma = \sigma_0$

(C) $H : \theta = \theta_0, \sigma = \sigma_0$ (D) $H : \theta \neq \theta_0$

60. The mean of R in Runs test under H_0 with usual notations is given by

(A) $\frac{2m}{m+n} + 1$ (B) $\frac{2n}{m+n} + 1$

(C) $\frac{2mn}{m+n}$ (D) $\frac{2mn}{m+n} + 1$

61. The Gauss Markov theorem establishes that the G.L.S. estimator of β, $\hat{\beta} = \left(X^1 V^{-1} X\right)^{-1} X^1 V^{-1} Y$ is

(A) Unbiased estimator only

(B) Error free estimator

(C) Both (A) and (B)

(D) Best linear unbiased estimator

62. The log linear models are analogous to ANOVA models with

(A) Unequal number of observations

(B) Equal number of observations

(C) Multiple observations

(D) Both (A) and (B)

63. If $R_{1.23} = 1$, then

(A) At least one regression residual is non-zero

(B) The multiple linear regression equation of X_1 on X_2 and X_3 is considered as perfect for predictions

(C) All total correlations involving X_1 are zero

(D) All partial correlations involving X_1 are zero

64. In a logistic regression the *S*-shaped curve is not symmetrical about its

(A) Increase in point of inflection

(B) The curve increases rapidly

(C) The curve decreases

(D) Point of inflection

65. The independent variables in logistic regression are called as

(A) Variates

(B) Covariates

(C) Both (A) and (B)

(D) Logit

66. A square symmetric matrix A and its associated quadratic form is called positive definite if

(A) $x'Ax \leq 0$ for every 'x' not equal to the null vector

(B) $x'Ax > 0$ for every 'x' not equal to the null vector

(C) $x'Ax < 0$ for every 'x' not equal to the null vector

(D) $x'Ax \geq 0$ for every 'x' not equal to the null vector

67. If $A_1, A_2, \cdots, A_q$ are independently distributed with A_i distributed according to Wishart distribution $W(\Sigma, n_i)$ then

$A = \sum_{i=1}^{q} A_i$ is distributed according to

(A) $W\left(\Sigma, \sum_{i=1}^{q} n_i\right)$

(B) $W(\Sigma, n_i)$

(C) $W\left(\Sigma^{-1}, \sum_{i=1}^{n} n_i\right)$

(D) $W\left(\frac{1}{\Sigma}, n_i\right)$

68. Let A and Σ be partitioned into q and p − q rows and columns

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}; \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

If A is distributed according to Wishart distribution $W(\Sigma, n)$ then A_{11} is distributed according to

(A) $W(\Sigma, n)$

(B) $W(\Sigma_{11}, n)$

(C) $W(\Sigma_{12}, n)$

(D) $W\left(\Sigma_{11}^{-1}, n\right)$

69. The function $a'x$ is known as

(A) Multiple Discriminant Analysis

(B) Linear Logistic Regression

(C) Linear Discriminant Function

(D) Logistic Discrimination

70. If the entries in Rows of a Latin square are same as its columns, the Latin square is called

(A) Conjugate

(B) Self conjugate

(C) Orthogonal

(D) Symmetric

71. The method of confounding is a device to reduce the size of

(A) Experiments

(B) Replications

(C) Blocks

(D) All the above

72. The systematic sampling and S.R.S. shall give estimates of equal precisions if the inter class correlation between the units of the same systematic sample, from a population of size N and a sample of size n, is equal to

(A) $\frac{1}{N-n}$ (B) $\frac{-1}{n-1}$

(C) $\frac{-1}{N-n}$ (D) $\frac{-1}{N-1}$

73. Suppose f(x) is a density on $(0, \infty)$ with distribution function f (x) which is DFR then the function g (x) = log f(x) is

(A) Convex

(B) Concave

(C) Constant

(D) The function value is 1

74. In which method we use the formula $\text{Min}\left\{X_{b_i} : X_{b_i} < 0\right\}$ to obtain the learning variable

(A) Dual Simplex

(B) Duality Problem

(C) Big M-method

(D) Simplex Method

75. If the arrival rate is 3 per hour and service rate is 6 per hour then the traffic intensity ρ equals to

(A) 6 (B) 3

(C) 2 (D) 0.5